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Polymer dispersed liquid crystal droplets: methods of calculation of optical characteristics

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The extinction efficiency factor and the angular distribution of radiation for polymer dispersed liquid crystal (PDLC) droplets have been calculated in the new discrete-dipole approximation (DDA) and compared with the data obtained in the commonly used Rayleigh–Gans approximation (RGA) and in the anomalous-diffraction approximation (ADA). The errors of calculations by the above mentioned methods for droplets with a homogeneous and for droplets with a radial director configuration are considered. The results of calculations of the optical properties of droplets with a radial and with a bipolar director configuration in the DDA are presented. Based on the results obtained the potentialities of the above methods for describing the scattering by liquid crystal droplets are illustrated.

1. Introduction

The application in optical devices of a liquid crystal (LC) material in the form of individual droplets embedded in a polymer matrix—polymer dispersed liquid crystal (PDLC) droplets—is attractive as a promising method for creating a number of electro-optical elements with high qualitative characteristics. The electro-optical parameters of such systems depend on the method of their manufacture [1–3]. They are determined by the size, shape and director configuration of the droplets, their concentration and orientation, the refractive index of the polymer and the droplets, etc. The choice of a particular combination of these parameters can considerably enhance or weaken the manifestation of the effect on which the LC devices are based. Therefore, an interesting problem is the theoretical description of structures based on LC composites with the aim of predicting the characteristics of optical devices and optimizing their parameters.

In terms of optics, the LC composite is a densely packed medium of discrete scatterers (the volume concentration can reach 80% [1]) whose main properties are determined by both the peculiarities of scattering from an individual droplet and the considerable influence of multiple scattering of waves from a system of droplets [4–10].

The optical properties of individual LC droplets and the influence on them of their structure and form are determining factors in describing the optical characteristics of LC composite-based devices. The description of such an object as an LC droplet is not easy because of peculiarities such as the non-sphericity of the form, the presence of the internal and the surface structure of the director and its fluctuations in the droplet, etc. At the same time, the LC droplet can be considered as an optically 'soft' object in the transparency mode and in the scattering mode. Therefore, to describe the optical properties of droplets, well known methods such as the Rayleigh–Gans approximation (RGA) and the anomalous-diffraction approximation (ADA) [11] are traditionally used. For small optically 'soft' scatterers with internal material anisotropy, the RGA is considered in [12–14], and ref. [15] considers the ADA for large optically 'soft' anisotropic droplets. Based on these approximations for spherical droplets and a number of simple configurations of the director in the droplet, analytical formulae have been obtained.

It is impossible to describe analytically configurations that are more complicated; for this, numerical calculations are needed. Recently a method has been developed for calculating characteristics of complex particles based on the discrete-dipole approximation (DDA) [16, 17]. It consists in presenting particles in the form of a system of dipoles, which permits, at an appropriate breakdown, a result to be obtained that is fairly close to the exact result for scatterers of practically any configuration and form. At the same time an essential disadvantage of this method is the large amount of computer resource needed for the calculation for both complex and simple particles,

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as well as the rapid increase in such resources needed with increasing size of scatterers. Therefore, this method can only be used for scatterers that are not very large. At present software for the calculation in the DDA for a number of scatterer forms is available (DDSCAT—Draine, Flatau, ftp://astro.princeton.edu/draine/scat/ddscat/ver5a), but it is not intended for the calculation of the characteristics of LC droplets for which each elementary dipole should have its own permittivity tensor, in the general case non-diagonal in a given system of coordinates. In the present work, we used the program DDSCAT modified by us for calculation on the DDA basis of the optical properties of LC droplets whose director configuration is determined by specifying the field of angles of rotation of the directors in the droplet bulk.

Approximate methods of description have such undoubted advantages as relative simplicity and speed of calculation. At the same time, they give some systematic error which is not always small and which can only be controlled outwith the method. The size of LC droplets R is comparable to the radiation wavelength λ. It includes droplets where it is possible to use the RGA (submicrometer sized droplets) and ADA (supramicrometer sized droplets). The size parameter $x \equiv 2\pi R/\lambda$ for light in a vacuum lies in a fairly wide range—from 0·5 to 50 as a rule. Therefore questions arise about the possibility and exactness of using both the RGA and the ADA for the intermediate range of droplet sizes, as well as about the value of the systematic error of these methods and the influence of the scatterer material anisotropy on the exactness of the approximate description. In most cases this question is resolved by using known conditions (for the RGA $2kR|n-1| \ll 1$, $n \approx 1$, for the ADA $kR \gg 1$, $n \approx 1$ [11], where k is the wave number in a vacuum, R is the droplet radius, n is the refractive index of the droplet). Note that for the above range of sizes of scatterers, this interpretation is diffuse and more detailed refinements of both the applicability of these approximations and the question of exactness are needed.

Sufficiently detailed comparisons of exact calculations according to Mie with approximate calculations in the RGA and the ADA for homogeneous objects have been made in [18–21]. The presence of anisotropy in the droplet, when the refractive index for different waves in the droplet can be both greater and smaller than unity, can considerably influence the exactness of the description of scattering by approximate methods. In addition, it is not clear how to interpret the condition of an infinitesimal refractive index of such a scatterer.

In the present work, the applicability of the above-mentioned approximate methods is considered, based on a comparison of the RGA and ADA with the DDA. As the objects for comparison, we have used spherical scatterers with a radial arrangement of directors [22] as characteristic of the arrangement of directors in a nematic LC droplet for which analytical formulae have been developed to calculate its properties. On the one hand, these droplets are similar to well-studied homogeneous scatterers with respect to spherical symmetry, but at the same time they differ considerably because of the existence of the internal structure of the anisotropic liquid crystal. We think that for other director configurations and droplet forms found in LC composites [1, 2], the conditions for RGA and ADA applicability, which are determined by certain effective values of the size and refractive index of the droplet and indirectly by its structure and form, will not change considerably.

Section 2 gives the formulae and methods for the RGA, ADA and DDA in accordance with which the calculations were made. Section 3 presents the results of calculations for spherical homogeneous scatterers with parameters approximately corresponding to LC droplets. They permit comparison of the results of the DDA with exact calculations according to Mie. Section 4 presents results of the calculation of the errors of the approximate methods (RGA and DDA) as compared with the DDA for a number of characteristic parameters of LC droplets with a radial director configuration. Some data on the calculation in the DDA for radial and bipolar configurations are also given.

2. Basic equations

Let us consider an LC scatterer placed in a polymer with refractive index n_m, in the lightwave field $\mathbf{E}^{inc} = \mathbf{e}_0 E_0 \exp(i\mathbf{k}_0\mathbf{r})$, where $\mathbf{e}_0$ is the unit vector of polarization. Let us denote the wave vector in the direction of wave scattering by $\mathbf{k}$, with $\mathbf{k}_s \equiv \mathbf{k} - \mathbf{k}_0$ as the scattering vector and $\mathbf{n}_0 \equiv \mathbf{k}_0/k_0$, $\mathbf{n} \equiv \mathbf{k}/k$. We write the scattered field normally as:

$$\begin{pmatrix} E_\parallel \\ E_\perp \end{pmatrix} = \frac{\exp(ikr)}{-ikr} \begin{pmatrix} S_2 & S_3 \\ S_4 & S_1 \end{pmatrix} \begin{pmatrix} E_\parallel^{inc} \\ E_\perp^{inc} \end{pmatrix} \qquad (1)$$

where $E_\perp$, $E_\parallel$ are the vector $\mathbf{E}$ components perpendicular and parallel to the scattering plane, respectively. We shall use the relative refractive index, so assuming $n_m = 1$. Accordingly, the further wave vector $k \equiv 2\pi/\lambda$ and the size parameter $x \equiv 2\pi R/\lambda$, where λ is the wavelength in the polymer matrix. Let us assume the liquid crystal is uniaxial. In this case, its permittivity tensor, when reduced to its diagonal form, has two coinciding values: $\hat{\varepsilon} = \mathrm{diag}(n_0^2, n_0^2, n_e^2)$—($n_0$ is an ordinary and n_e an extraordinary refractive index).

Only for a small and sufficiently 'soft' scatterer ($2kR|n-1| \ll 1$, $n \approx 1$, as here n is the effective refractive

index and R is the droplet radius) in the RGA we have [12]:

$$\hat{S} = \frac{-\mathrm{i}k^3}{4\pi} \int (\hat{\varepsilon} - 1) \exp(-\mathrm{i}\mathbf{k}_s\mathbf{r})\, \mathrm{d}^3\mathbf{r}. \tag{2}$$

For the case of the axially symmetric scatterer under consideration, the non-diagonal values of the scattering matrix are zero:

$$\hat{S} = \mathrm{diag}(S_2, S_1). \tag{3}$$

For the scattering efficiency factor we have

$$Q_{\mathrm{sca}} = \left(\frac{\pi}{k}\right)^2 \int (|S_1|^2 + |S_2|^2) \sin\delta\, \mathrm{d}\delta \tag{4}$$

where δ is the polar angle of scattering.

In the anomalous diffraction approximation, the droplet is assumed to be 'soft' and fairly large $kR \gg 1$, $n \approx 1$ [11]. Then, in accordance with [15], the scattering matrix $\hat{S}$ will be written in the form

$$\hat{S} = \frac{2\pi\mathrm{i}}{\lambda^2} \int [\hat{p}(\mathbf{r}) - 1] \exp(-\mathrm{i}\mathbf{k}\mathbf{r})\, \mathrm{d}^2\mathbf{r} \tag{5}$$

where the tensor $\hat{p}(\mathbf{r})$ describes the phase shift and the rotation of the polarization plane of the beam passing through the point $\mathbf{r}$ of the scatterer projection on the plane perpendicular to the direction of wave propagation. The expressions for $\hat{p}(\mathbf{r})$ and the results of integration in equation (5) for a number of director configurations were obtained in [15]. These relations were used by us in further calculations.

The discrete-dipole approximation is based on the representation of the scatterer in the form of an array of discrete dipoles with corresponding properties. Each dipole is characterized by its position and polarizability, the choice of which is a non-trivial task. In accordance with [17], the problem of finding the polarization of elementary dipoles is reduced to the solution of the following system of linear equations:

$$P_j = \hat{\alpha}_j \left[E_j^{\mathrm{inc}} + \sum_{k \neq j} A_{jk} P_k \right]. \tag{6}$$

Here $\hat{\alpha}_j$ is the j-th dipole polarizability (see [16, 17] for expressions for $\hat{\alpha}_j$), the term $\hat{A}_{jk}P_k$ represents the field induced by the k-th dipole with polarization P_k on the j-th dipole.

As a rule, the solution of the system of equations (6) is reduced to iterations whose physical meaning consists in taking into account a greater and greater multiplicity of scattering from the system of N elementary dipoles. Proceeding from the known values of polarization on each elementary dipole for the matrix elements S_r, $r = 1, 2, 3, 4$, we can write [17]:

$$S_r = \sum_{j=1}^{N} \frac{\mathrm{i}k^3 P_j \nu_r}{E_0} \exp(-\mathrm{i}\mathbf{k}\mathbf{r}_j) \tag{7a}$$

$$r = 1\text{: } \nu_1 \equiv \mathbf{n} \times \mathbf{n}_0,\ \mathbf{e}_0 = \mathbf{n} \times \mathbf{n}_0 \tag{7b}$$

$$r = 2\text{: } \nu_2 \equiv \mathbf{n} \times (\mathbf{n} \times \mathbf{n}_0),\ \mathbf{e}_0 = \mathbf{n}_0 \times (\mathbf{n} \times \mathbf{n}_0) \tag{7c}$$

$$r = 3\text{: } \nu_3 \equiv -\mathbf{n} \times (\mathbf{n} \times \mathbf{n}_0),\ \mathbf{e}_0 = \mathbf{n} \times \mathbf{n}_0 \tag{7d}$$

$$r = 4\text{: } \nu_4 \equiv -\mathbf{n} \times \mathbf{n}_0,\ \mathbf{e}_0 = \mathbf{n}_0 \times (\mathbf{n} \times \mathbf{n}_0). \tag{7e}$$

For the j-th dipole polarizability we have

$$\hat{\alpha}_j = \hat{M}_j^{-1} \hat{\alpha} \hat{M}_j, \tag{8}$$

where $\hat{\alpha}$ is the diagonal tensor of polarizability used in [16, 17].

$$M_j = R_x(\alpha_j) R_y(\beta_j) R_z(\gamma_j), \tag{9}$$

where α_j, β_j, γ_j are rotation angles of the j-th dipole about the axes x, y, z, respectively; $\hat{R}_x(\alpha)$, $\hat{R}_y(\beta)$, $\hat{R}_z(\gamma)$ are matrices of rotation about the axes.

For the radial director configuration for these angles we have used

$$\alpha_j = \arctan(z_j, y_j) \tag{10a}$$

$$\beta_j = 0 \tag{10b}$$

$$\gamma_j = \arccos\left[\frac{x_j}{(x_j^2 + y_j^2 + z_j^2)^{1/2}} \right]. \tag{10c}$$

The polarizability tensor $\hat{\alpha}$ can be obtained in accordance with the formulae given in [16, 17] proceeding from the permittivity tensor $\hat{\varepsilon} = \mathrm{diag}(\varepsilon_e, \varepsilon_0, \varepsilon_0)$ where x_j, y_j, z_j are the j-th dipole coordinates relative to the scatterer centre.

For bipolar droplets, we used parametrization in which the molecules are oriented along a series of elliptical lines lying on a series of concentric nested ellipsoids with a common axis. In this case, the polarizability tensor $\hat{\alpha}$ is obtained by rotating the permittivity tensor $\hat{\varepsilon} = \mathrm{diag}(\varepsilon_0, \varepsilon_e, \varepsilon_0)$ by following:

$$\alpha_j = \arctan(z_j, y_j) \tag{11a}$$

$$\beta_j = 0 \tag{11b}$$

$$\gamma_j = \frac{\pi}{2} - \arctan[\, x_j (y_j^2 + z_j^2)^{1/2}, R^2 - x_j^2]. \tag{11c}$$

For the scattering efficiency factor equal to the attenuation efficiency factor in the case of non-absorbing scatterers we have [17]

$$Q_{\mathrm{sca}} = \left(\frac{4}{xE_0}\right)^2 \sum_{j=1}^{N} \mathrm{Im}(k^3 P_j E_j^{\mathrm{inc}}). \tag{12}$$

As the quantity characterizing exactness when comparing the two methods of calculation in § 3 and § 4, we

have chosen the relative error: for the factor of scattering efficiency Q^{ref}, the quantity

$$\Delta_Q \equiv \frac{Q^{\text{ref}} - Q}{Q}. \tag{13}$$

As the function characterizing the angular structure of the radiation, we have chosen the dimensionless value I:

$$I \equiv \frac{1}{2}(|S_1|^2 + |S_2|^2). \tag{14}$$

This dimensionless value is related to the differential scattering cross section $\mathrm{d}\sigma/\mathrm{d}\Omega$ for natural light by the relation

$$I = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} k^2. \tag{15}$$

Here k is the wave number in a vacuum. Then the calculated value I^{ref} is characterized by the error

$$\Delta_I \equiv \frac{I^{\text{ref}} - I}{I}. \tag{16}$$

By Q and I are meant the values obtained as being more exact (in §3—from the calculation according to the Mie theory, in §4—in the DDA).

3. The homogeneous scatterer

Figure 1 gives some results of comparing the DDA and the approximate methods RGA and ADA with exact calculations (according to the Mie theory) for spherical homogeneous scatterers. We chose the refractive index $n = 1{\cdot}11$ for the homogeneous droplet as an illustration for relatively 'hard' LC droplets [23].

As can be seen, for the scattering efficiency factor Q_{sca}, depending on the droplet size, the accuracy of calculation by the DDA method is fairly high within the scope of applicability of the chosen breakdown [17] (at a breakdown of $29 \times 29 \times 29$, approximately up to $x = 24$ with an accuracy better than 5% and up to $x = 11$ with an accuracy better than 1%); finer breakdown permits description of larger scatterers (at a breakdown of $37 \times 37 \times 37$, approximately up to $x = 34$ with an accuracy better than 5% and up to $x = 22$ with an accuracy better than 1%). We have not noticed any possible deviations from this regularity associated with the accumulation of errors with increasing number of both elementary dipoles and iterations for finding the distribution of dipole polarization due to multiple scattering in the calculations ($n \leqslant 1{\cdot}2$, breakdown of up to $40 \times 40 \times 40$ dipoles) for Q_{sca}.

One can see that for small particle sizes, there is a difference between the RGA and Mie calculations. This arises from the assumption in the RGA that the particle refractive index is close to unity. The larger the value, the bigger the difference. In the case presented, the discrepancy in less than 15%.

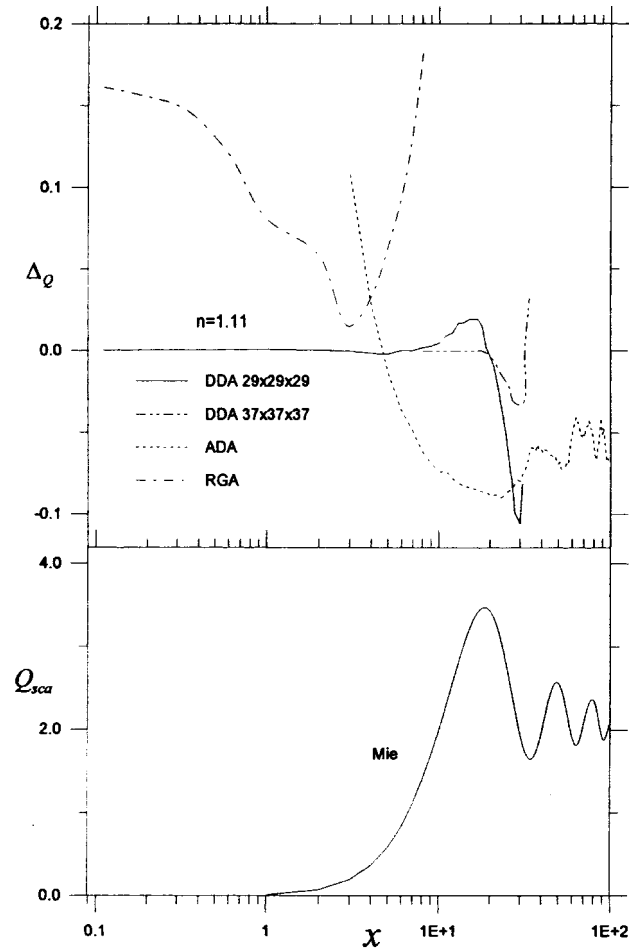


Figure 1. The scattering efficiency factor Q_{sca} (according to the Mie theory) as a function of size parameter x for a homogeneous sphere with $n = 1.11$ and the error of its calculation Δ_Q for the DDA, RGA, and ADA.

Figure 2 shows the results of the calculations for the angular structure of scattering of 'soft' homogeneous droplets with $x = 5$, $n = 1{\cdot}05$. As can be seen, the RGA and DDA approximations are not exact enough to describe the angular structure of scattering of such medium sized droplets. The RGA gives an accuracy of 5% for the angular distribution up to angles of scattering less than 30°. The ADA gives an error of 10% up to angles of about 20°. For the break-down $29 \times 29 \times 29$, the ADA gives an accuracy better than 5% for the radiation structure up to angles of about 90°, and up to 160° with an accuracy of 10%. In the rear hemisphere, the accuracy of the description is worse, but nevertheless the error does not exceed 20% at the above breakdown. Note that the contribution of scattering at large angles

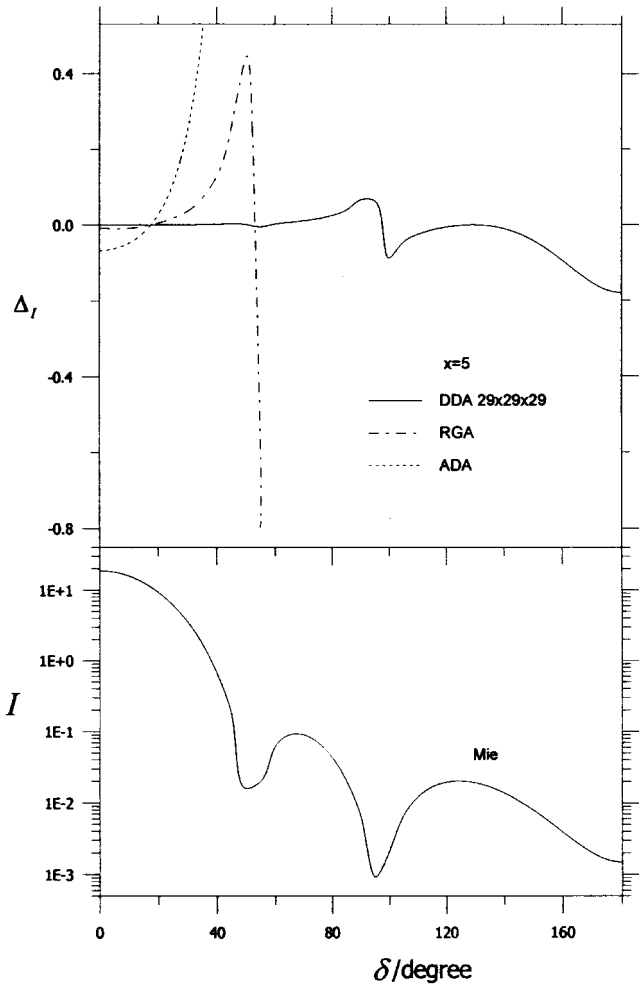


Figure 2. The dependence of I (according to the Mie theory) on the polar angle δ for a homogeneous sphere with $x = 5$, $n = 1{\cdot}05$ and the error of its calculation Δ_I for the DDA, RGA and ADA.

to the light flux decay is smaller by a few orders of magnitude.

For 'harder' droplets, the RGA and ADA errors increase. For the DDA the calculation time (number of iterations) increases, and there is no direct relationship between the refractive index of the droplet and the accuracy of calculation in the DDA.

Detailed analysis of DDA errors for homogeneous scatterers has been carried out in [16, 17]. Our comparisons for homogeneous scatterers show that the DDA has a considerably better accuracy than the above mentioned approximations over a range of parameters characteristic of LC droplets. The exception is the region of scattering at large angles where the accuracy is limited. From the point of view of the DDA, there is no essential difference between the errors of the system for isotropic and anisotropic elementary scatterers. Therefore, the DDA can be used to estimate the RGA and ADA errors for anisotropic LC droplets. On this basis, § 4 considers the question of the exact description of a spherical scatterer with a radial director configuration by the RGA and ADA approximations with respect to the DDA.

4. The liquid crystal droplet

Figures 3–6 give the results of the error calculations of the scattering efficiency factor and angular structure of the radiation for spherical liquid crystal droplets with a radial structure of the director. The extraordinary refractive index of the liquid crystal in the droplet was chosen in the range from a 'soft' $n_e = 1{\cdot}08$ to a 'hard' droplet $n_e = 1{\cdot}2$. The normal refractive index had the values $n_0 = 0{\cdot}97$ and $0{\cdot}99$.

Figure 3 shows plots of the error of calculating the scattering efficiency factor for four values of the extraordinary refractive index of the droplet $n_e = 1{\cdot}08$, $1{\cdot}11$,

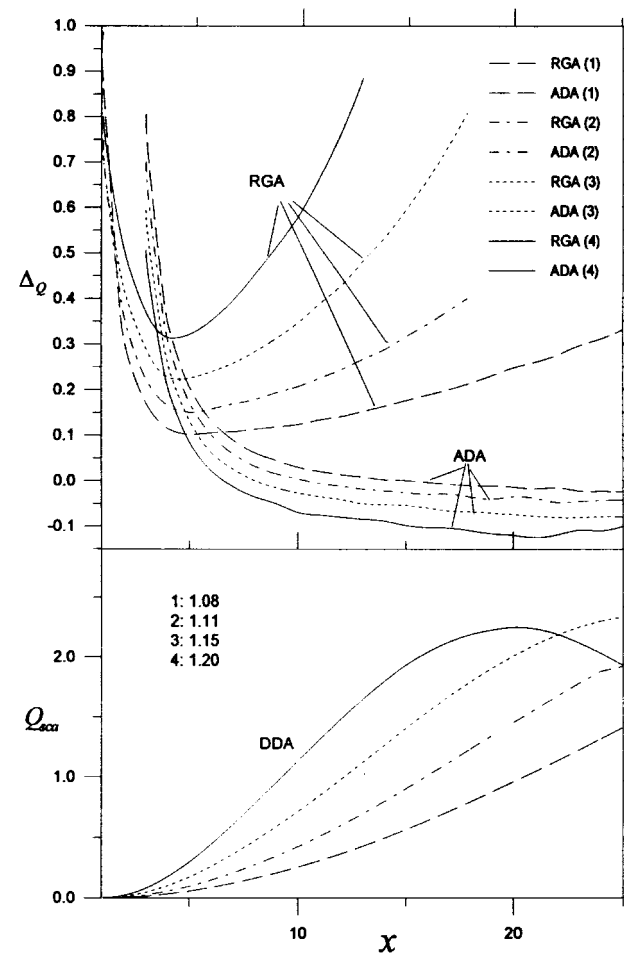


Figure 3. The scattering efficiency factor Q_{sca} (DDA) as a function of size parameter x for a spherical LC droplet with radial director configuration for $n_0 = 0{\cdot}97$, $n_e = 1{\cdot}08(1)$, $1{\cdot}11(2)$, $1{\cdot}15(3)$, $1{\cdot}20(4)$ and the error of its calculation Δ_Q for the RGA and ADA.

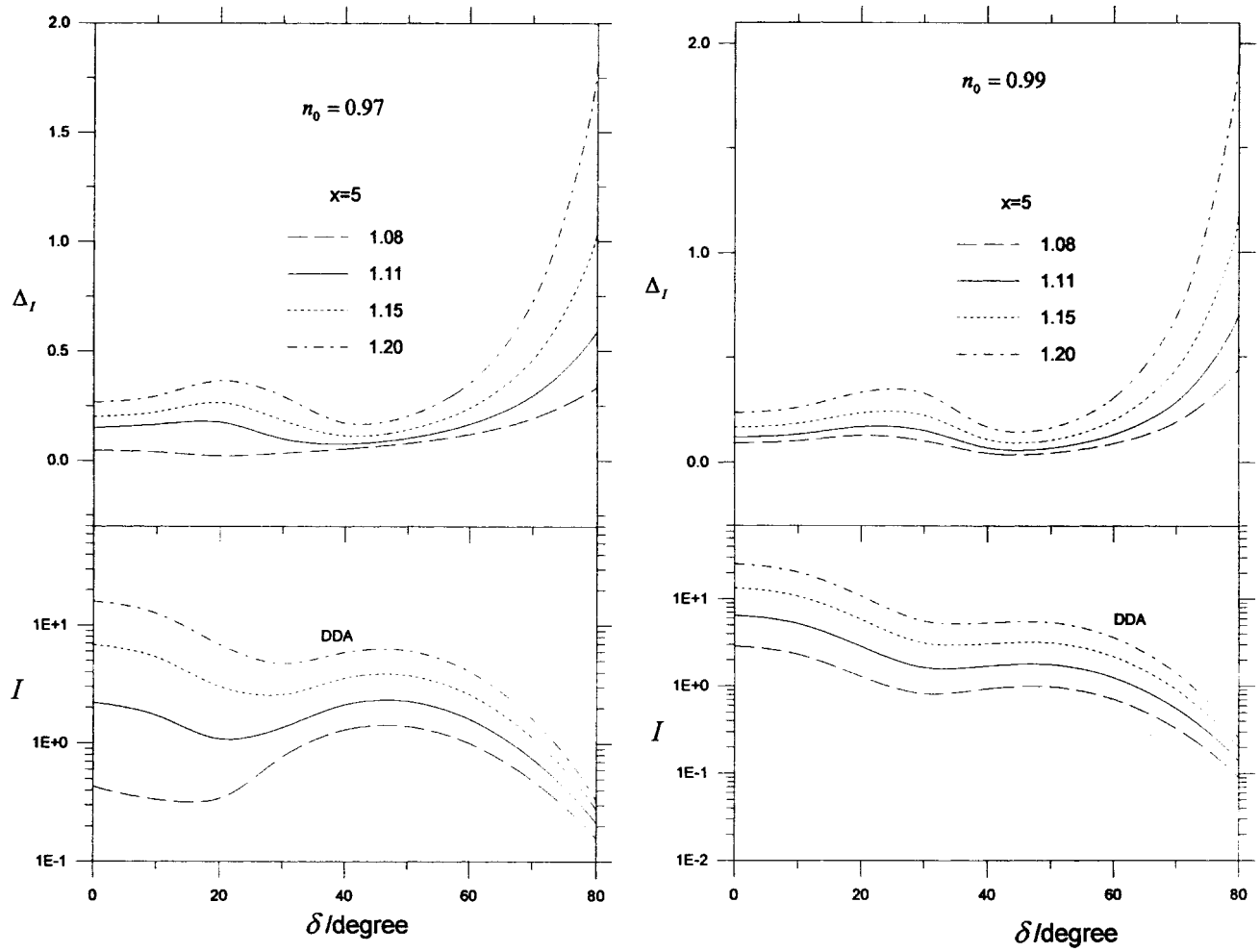
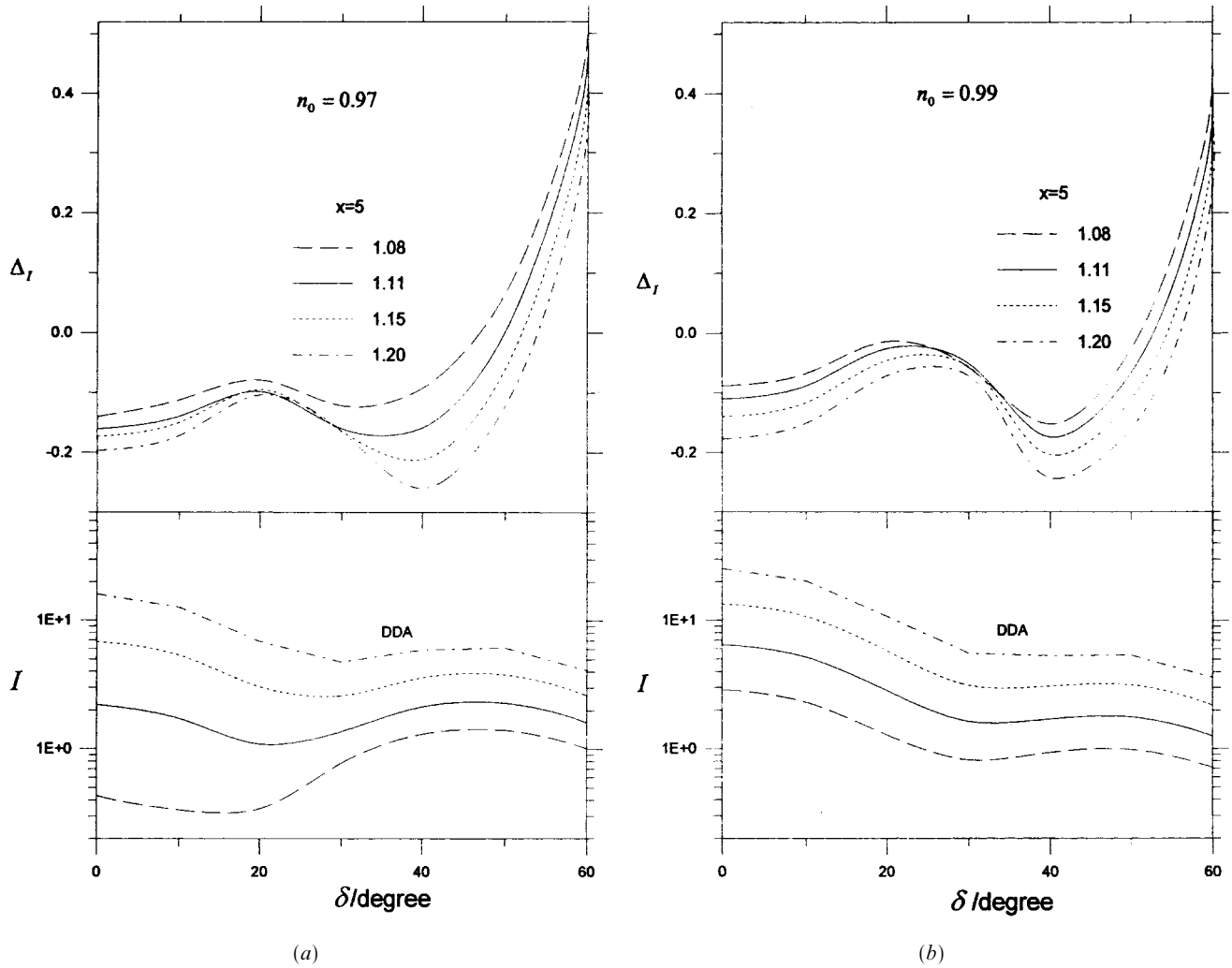


Figure 4. The dependence of I (DDA) as a function of the polar angle δ for a spherical LC droplet with radial director configuration at $x = 5$, $n_0 = 0{\cdot}97$ (*a*) and $n_0 = 0{\cdot}99$ (*b*) and the error of its calculation Δ_I, for the RGA; n_e values are listed.

1·15, 1·2. As can be seen, the ADA error, as a whole, is similar to that which can be expected from the analysis of the ADA error for homogeneous scatterers. As with the homogeneous scatterer, in the range of changes in the refractive index, from a scatterer size of about $x = 10$, the anomalous diffraction approximation can be used to describe the LC droplet with a moderate error (which in our case does not exceed 15%). Note that within the limits of the parameters considered by us, the position of this range depends weakly on the refractive index. The ADA can be used, with a model accuracy of 25%, even from the size of the droplet $x = 5$ for both Q and the angular structure of scattering (see figure 5). Comparison of the error plots for Q with those for homogeneous scatterers shows that the behaviour of the radial structure droplet for the ADA is in general the same as that of the considerably 'harder' scatterer—the homogeneous one with the refractive index $n_{\text{eff}} = 2n_0 + n_e/3$. Therefore the error plot for the homogeneous scatterer in figure 1 (refractive index $n = 1{\cdot}11$) is analogous to the plot with $n_e = 1{\cdot}2$ for the droplet with radial configuration. As in the case of homogeneous scatterers, the maximum angle to which the ADA approximation can describe the angular structure of radiation does not exceed 30°, and this angle slightly depends on the scatterer parameters.

Such a picture of changes in the ADA error for LC droplets can be explained by the existence of changes in the refractive index on the path of the beam inside the LC droplet. This leads to a worsening of the conditions for the ADA applicability as compared with the homogeneous scatterer. At the same time, for the radial configuration these changes are not as great as for some other configurations of the director realized in the LC droplet [2]. Therefore, more significant errors of ADA descriptions are possible.



(*a*) (*b*)

Figure 5. The dependence of I (DDA) as a function of the polar angle δ for a spherical LC droplet with radial director configuration at $x = 5$, $n_0 = 0{\cdot}97$ (*a*), $n_0 = 0{\cdot}99$ (*b*) and the error of its calculation Δ_I, for the ADA; n_e values are listed.

The transfer from isotropic to anisotropic droplets (see, for example, figures 3 and 4) for the RGA, unlike the ADA, significantly affects the accuracy of this approximation. Although the position of the error minimum for them is shifted toward large scatterers, as with 'softer' droplets, its value increases significantly compared with homogeneous scatterers, as with 'harder' droplets. Therefore the error value for droplets with a radial structure already exceeds 10% even for the 'softest' ($n_e = 1{\cdot}08$) particles, although with the homogeneous scatterer with refractive index $n = 1{\cdot}11$, the error is no more than 2%. This points to the growth of the influence of scattering multiplicities higher than the first one on the properties of LC droplets as compared with homogeneous scatterers in the range of sizes characteristic of the RGA.

Note the non-characteristic high maximum of homogeneous scatterers on the plot of the quantity I as a function of angle (figures 4 and 5) at $\delta \approx 50°$. For $n_e = 1{\cdot}08$ and $n_e = 1{\cdot}11$, figures 4(*a*) and 5(*a*), its value exceeds the value of I at $\delta \approx 0°$. Such a dependence of the angular structure is absent for homogeneous and the appearance of a high maximum at $\delta \approx 50°$ is due to the peculiarities of scattering from the type of inhomogeneous droplets under consideration. Such a picture of scattering by a droplet with radial configuration was considered in [15] and attributed to the analogy of the scattering from the radial configuration droplet with the scattering from an annular screen. Results are given in figures 6(*a*) and 6(*b*) for a similar set-up to that in figures 5(*a*) and 5(*b*), but with $x = 15$.

Figure 7 shows the results obtained in the DDA for radial and bipolar director configurations. Figure 7(*a*) gives data for the normal refractive index $n_0 = 0{\cdot}97$ and figure 7(*b*) for $n_0 = 0{\cdot}99$. For the bipolar configuration, the light radiation is incident on the droplet along its axis.

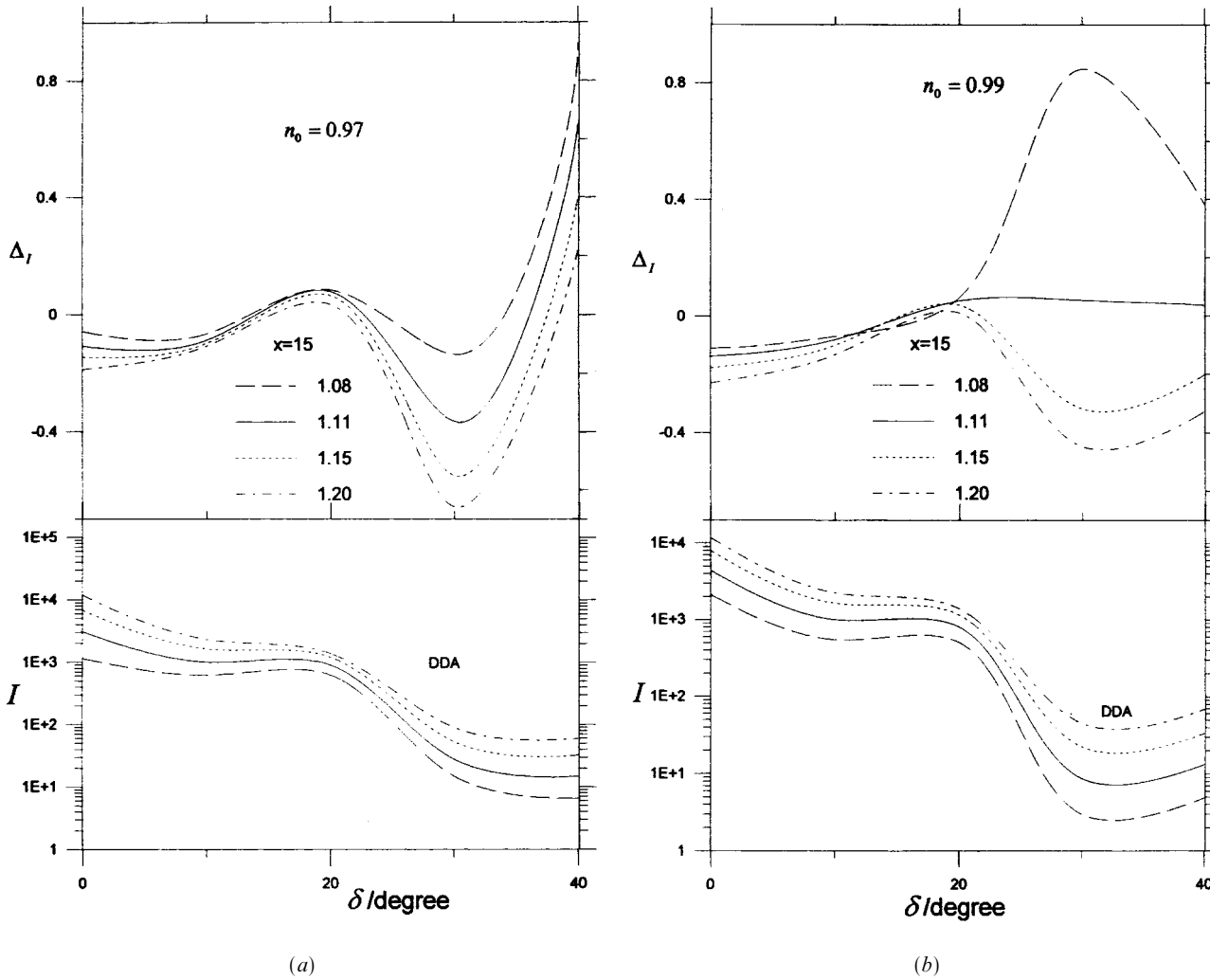


Figure 6. The dependence of I (DDA) as a function of the polar angle δ for a spherical LC droplet with radial director configuration at $x = 15$, $n_0 = 0{\cdot}97$ (*a*), $n_0 = 0{\cdot}99$ (*b*), and the error of its calculation Δ_I for the ADA; n_e values are listed.

As seen from the plots, for the bipolar droplet high maxima in the scattering at large angles, which are not characteristic of homogeneous droplets, are also observed. They are especially pronounced for the bipolar structure on the plots with $n_0 = 0{\cdot}97$. On the plots with $n_0 = 0{\cdot}99$, the values of these maxima diminish relative to the value for the forward direction peak. The same occurs when n_0 is above some value which corresponds to the choice of the polymer refractive index close to the effective refractive index of the droplet. In this case, the dependence $I(\delta)$ is as a rule other than that for the homogeneous scatterer. At a deviation from this value, towards larger or smaller values, the droplet, from the viewpoint of the angular structure of scattering, becomes similar to the homogeneous case (with the refractive index greater or smaller than unity). Therefore, the change in the form of the angular structure on the plots in figures 7(*a*) and 7(*b*) involves the reduction or rise of the forward direction scattering peak. As a rule, the reduction, such as shown in figure 7(*a*) for the radial structure and in figure 7(*b*) for the bipolar structure, corresponds to a decrease in the extinction efficiency factor, i.e. to an increase in the PDLC layer transparency.

For the radial director configuration, the value of the effective refractive index is obviously closer to the extraordinary index in comparison with the bipolar structure. Therefore, the value of n_0 corresponding to the choice of the refractive index of the medium close to the effective refractive index of the droplet is smaller than for the bipolar director configuration. This peculiarity is illustrated by figures 7(*a*) and 7(*b*). Then, while for the radial configuration the plots of the quantity of scattering as a function of angle differ most greatly from

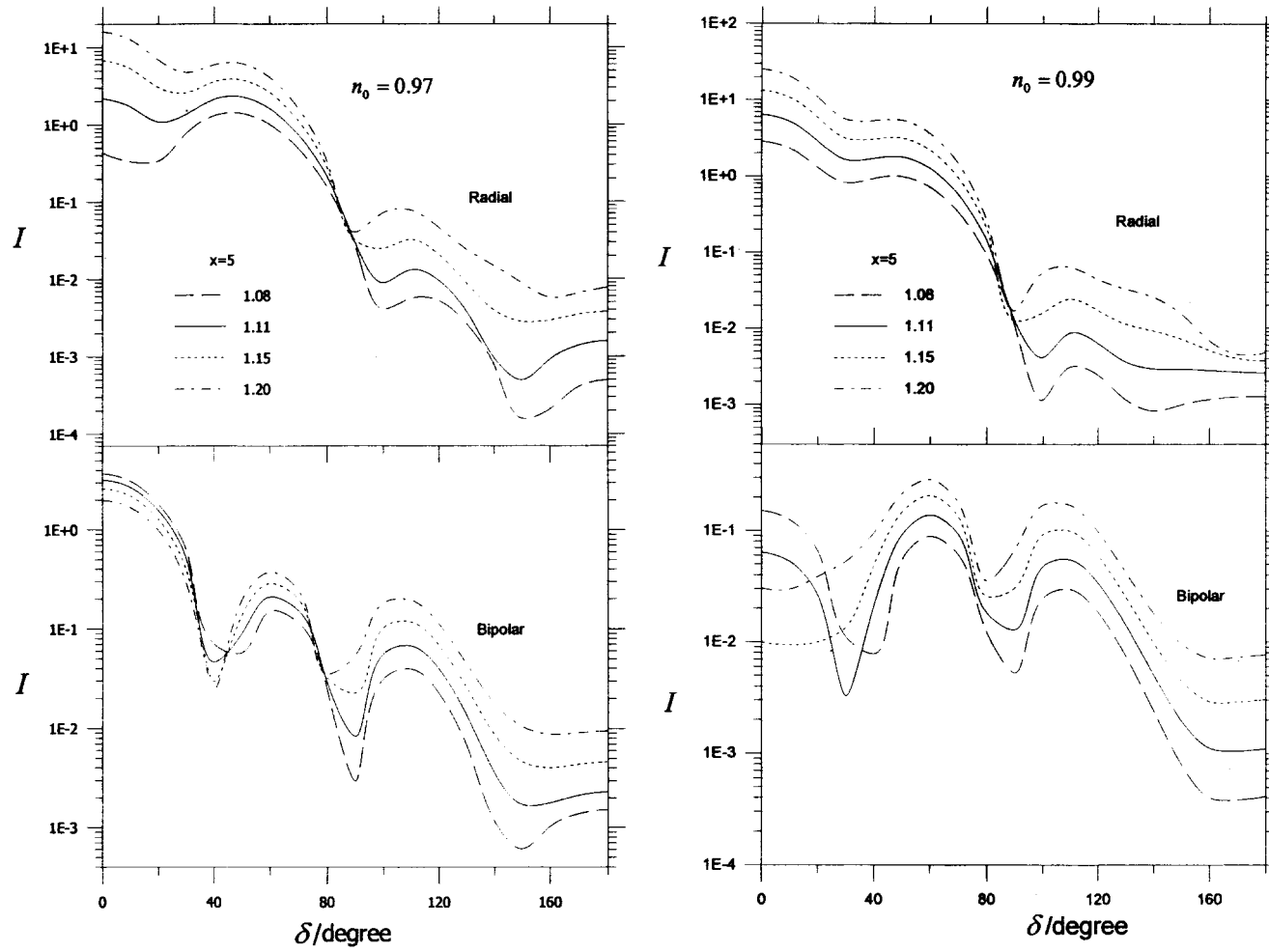


Figure 7. The dependence of I (DDA) as a function of the polar angle δ for bipolar (bottom) and radial (top) director configurations of a spherical LC droplet at $x = 5$, $n_0 = 0{\cdot}97$ (*a*), $n_0 = 0{\cdot}99$ (*b*) and $n_e = 1{\cdot}08(1)$, $1{\cdot}11(2)$, $1{\cdot}15(3)$, $1{\cdot}20(4)$. For the bipolar director configuration the wave vector of the incident light coincides with the droplet director.

that for the homogeneous droplet at $n_0 = 0{\cdot}97$, for the bipolar droplet the greatest difference is at $n_0 = 0{\cdot}99$.

In general, the same result for LC droplets is also obtained in the RGA [12]. Two kinds of terms enter into the RGA formulae for the angular structure of scattering: these are the terms proportional to the parameter $\zeta \equiv \mathrm{Tr}(\varepsilon)/3 - 1$ and to the parameter $\eta \equiv (\varepsilon_e - \varepsilon_0)/3$. The terms proportional to ζ correspond to the homogeneous droplet with permittivity $\varepsilon = \zeta$, and the terms proportional to η are characteristic of the droplet with a structure. In accordance with the RGA, if $\zeta = 0$, then only that term characteristic of the droplet with a structure is not equal to zero in the formula describing the angular structure of the radiation. In addition, the condition $\zeta = 0$ defining the maximum transparency of the sample does not depend on the droplet structure. However, as can be seen from the analysis of figures 7 (*a*) and 7 (*b*) and analogous plots for different director configurations, this simple condition is certainly a simplification specific for the RGA.

5. Conclusion

We have analysed the error for the radial director configuration in an LC droplet. Analysis of the error for the general case of director configuration is complicated from the point of view of the possibility of comparing results for a wide range of droplets. At the same time, we hope the results presented in this paper give, in general, answers to questions about peculiarities of applicability of the referred approximations for LC droplets. As can be seen from these, the DDA approximation can describe the scattering efficiency factor fairly exactly, including that portion of the range of

size parameters where neither RGA nor ADA gives a sufficiently accurate result. As would be expected, the DDA has an apparent advantage from the viewpoint of exactness of the description. Note that for the RGA approximation, in accordance with our results, there are difficulties concerning the description of anisotropic scatterers even in the range of parameters traditionally characteristic of the RGA. As with homogeneous scatterers, the ADA approximation cannot describe the angular structure of scattering for angles that are not small. In this region, there is no alternative to the DDA or similar more complicated approximations, which take into account in detail the structure of the incident wave scattering inside the droplet. At the same time, for the DDA itself, for large angles of scattering and large sizes of droplets, certain complications of computing are observed.

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